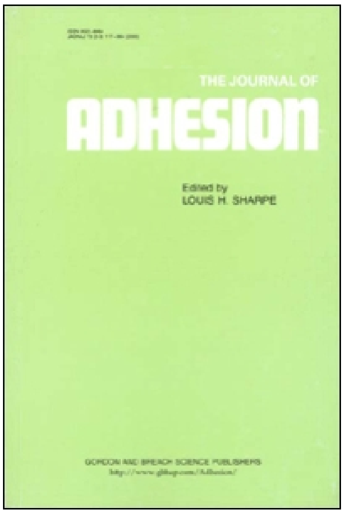

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Adherence and Rolling Kinetics of a Rigid Cylinder in Contact with a Natural Rubber Surface

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Equilibrium conditions of a rigid cylinder in contact with the flat and smooth surface of a natural rubber sample are studied using concepts of fracture mechanics, such as the strain energy release rate or the stress intensity factor. It is shown that an equilibrium contact area exists if the applied force per unit axial length is greater than a negative critical value, closely related to the cylinder radius and mechanical and superficial properties of the elastic solid. Due to the intervention of molecular attraction forces, of van der Waals type, a light cylinder rolls under an inclined rubber surface and it is displayed that the rolling speed is the same when the cylinder rolls upon the same inclined surface. It has been verified that if a flat rubber substrate, with an adequate length, is rotated at constant angular velocity, a steel cylinder rolls alternately upon and under the surface, unceasingly without falling down.

KEY WORDS Adherence; crack propagation; cylindrical punch; fracture mechanics; natural rubber; rolling friction.

INTRODUCTION

In previous works,[1,2] Maugis and Barquins have shown that the strain energy release rate G and its derivative $\partial G/\partial A$ with respect to the area of contact A allow one to scrutinize the equilibrium conditions and the stability of the contact between axisymmetrical punches and viscoelastic bodies. The edge of the contact area is assumed to be a crack tip which propagates in the interface, moving backward or forward as the applied force is increased or decreased. This approach has the advantage of enabling one to determine the elastic adherence force, to study the kinetics of crack propagation and to predict the evolution of the system whatever the geometry of contact and loading conditions.

The present study examines the adhesive contact between a long rigid cylinder and an elastic half-space. The well-known difficulties encountered when one attempts to determine the vertical elastic displacement of the cylinder under a specified normal load, do not allow us to calculate the strain energy release rate using an energy balance theory, as in previous works. In compensation, as suggested by Savkoor and Briggs[3] for their examination of the contact of spherical punches, G can be easily deduced from the stress intensity factor at the edge of the contact area of which the size is calculable from the classical theory of elasticity.

ADHESIVE CONTACT

Let us consider a long and rigid cylinder, with weight W, radius R and length l, in contact with the flat and smooth surface of an elastic solid, characterized by its Young's modulus E and its Poisson's ratio ν. Under the force P applied per unit axial length, $P = W/l$, the half-width b_H of the contact area, derived from the Hertz theory,[4] is:

$$b_H = \{4PR(1-\nu^2)/\pi E\}^{1/2} \quad (1)$$

However, as it was pointed out by Johnson *et al.*[5] in 1971 for contacts of spheres, Hertz's calculations do not take into account the attractive molecular forces (van der Waals's forces in the case of elastomers) which contribute to the increase of the contact area, so that the actual half-width b of the cylinder contact is greater than b_H.

Let us assume that the actual size of b results from the action of the apparent Hertz load $P_1 > P$ in the absence of surface energy effects; its value can be expressed by an equation similar to Eq. (1):

$$b = \{4P_1R(1-\nu^2)/\pi E\}^{1/2} \quad (2)$$

Then, at constant width one can decrease the load per unit axial length from the fictitious value P_1 to the actual value P by increasing surface energy. This procedure creates normal tensile stresses at the edge of the contact area similar to these provoked by the vertical displacement of a flat punch rectangular in shape, with the same length l as that of the cylinder and a width equal to $2b$, when the flat punch is submitted to the linear force $(P - P_1)$. Obviously, the previous procedure assumes that the degree of indentation of the rubber sample by the rigid cylinder is small, *i.e.*, the width of contact area is very small with respect to the cylinder length.

At the distance r of the longitudinal symmetry axis of the contact area, the normal tensile stress can be written:[6]

$$\sigma_z = (P_1 - P)/\pi(b^2 - r^2)^{1/2}$$

so that the stress intensity factor K_I at the edge of the contact area for crack propagation in mode I (opening mode) which is defined by:

$$K_I = \lim_{r \to b} [\sigma_z\{2\pi(b - r)\}^{1/2}]$$

is equal to:

$$K_I = (P_1 - P)/(\pi b)^{1/2}$$

Like any three-dimensional crack, the edge of the contact area is subjected locally to a plane strain state, so that the strain energy release rate G is linked to the stress intensity factor K_I by:

$$G = K_I^2(1 - \nu^2)/E$$

so, this rate can be expressed:

$$G = (1/2)(P_1 - P)^2(1 - \nu^2)/\pi E b \tag{3}$$

In this relation, on the one hand, the factor (1/2) is introduced in order to take into account the punch rigidity, and on the other hand, the force P_1 is linked to the actual half-width b by Eq. (2):

$$P_1 = \pi E b^2/4R(1 - \nu^2) \tag{4}$$

The equilibrium state of the contact system is defined by $G = w$ (Griffith's criterion), where w is the Dupré energy of adhesion, determined from the surface (γ_1, γ_2) and interface (γ_{12}) energies of solids 1 and 2 in contact by:

$$w = \gamma_1 + \gamma_2 - \gamma_{12}$$

so that the half-width b of the contact area and the load P, applied per unit axial length, are linked by the equilibrium relationship:

$$P = \pi E b^2/4R(1 - \nu^2) - \{2\pi E b w/(1 - \nu^2)\}^{1/2} \tag{5}$$

The equilibrium given by $G = w$ can be stable, unstable or neutral. A thermodynamic system under a given constraint is stable if the corresponding thermodynamic potential is minimum, *i.e.*, if its second derivative is positive. In a test at fixed load and temperature, the thermodynamic potential is the Gibbs free energy, and the stability is defined[1] by $(\partial G/\partial A)_p > 0$, where A is the contact area. The load P_c corresponding to the limit of stability, $((\partial G/\partial A)_p = 0)$, which may be called the elastic adherence force, represents the critical tensile force for which the spontaneous rupture of the contact area just begins.

For the adhesive contact between a rigid cylinder and an elastic half-space, the elastic adherence force P_c, per unit axial length, is equal to:

$$P_c = -3\{\pi E w^2 R/16(1 - \nu^2)\}^{1/3} = -3P_1 \tag{6}$$

and the corresponding half-width b_c of the ultimate equilibrium contact area is:

$$b_c = \{2wR^2(1 - \nu^2)/\pi E\}^{1/3} \tag{7}$$

Consequently, a rigid cylinder, of which the weight per unit axial length is smaller than the absolute value of P_c, remains in equilibrium contact **under** a flat elastic substrate, due to the intervention of surface energy effects.

Using the following normalisations for forces and lengths:

$$[P] = P/\{\pi E w^2 R/16(1 - \nu^2)\}^{1/3} \tag{8}$$

$$[b] = b/\{2wR^2(1 - \nu^2)/\pi E\}^{1/3} \tag{9}$$

the equilibrium relationship (Eq. (5)) takes the simple form:

$$[P] = [b]^2 - 4[b]^{1/2} \tag{10}$$

whereas Eq. (4), deduced from the Hertz theory, can be written:

$$[P] = [b]^2 \tag{11}$$

With the same normalizations, the reduced form of the strain energy release rate, $[G] = G/w$, is given by:

$$[G] = ([b]^2 - [P])^2/16[b] \tag{12}$$

Figure 1 illustrates, in reduced coordinates, the equilibrium relations for an adhesive contact (Eq. (10), heavy line) and for a non-adhesive Hertzian contact (Eq. (11), lower dotted line).

In order to verify the theoretical predictions, experiments were carried out using a smooth steel-cylinder with length $l = 20$ mm, radius $R = 5$ mm and weight $W = 122.5$ mN, in contact against the flat and smooth surface of a transparent unfilled natural rubber strip cured by dicumyl peroxide ($E = 0.9$ MPa, $\nu \simeq 0.5$), 2 mm thick, glued on a horizontal glass plate. Before contact, the surfaces of steel and rubber samples were wiped with an alcohol-soaked rag, dried with warm air and left, sheltered from dust, for 15 min for the equilibrium with room temperature to be reached. Then, the steel-cylinder was gently applied against the rubber surface with the help of a precision balance. The width of the contact area was measured in reflexion, through both the glass substrate and transparent rubber strip, in a microscope. Moreover, temperature (22°C) and humidity ratio (54%) were kept constant.

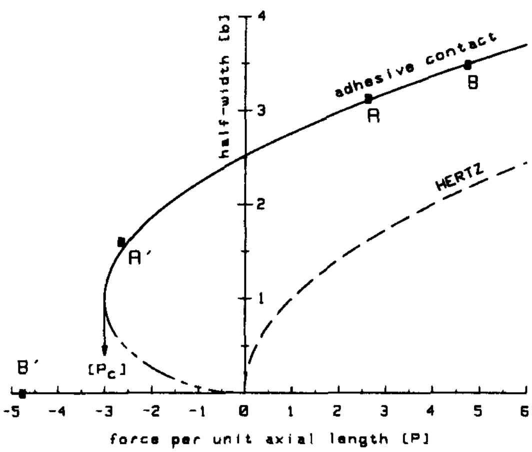

FIGURE 1 Half-width of the equilibrium contact area between a rigid cylinder and the smooth surface of an elastic solid as a function of the force per unit axial length, in reduced coordinates. For comparison, the classical Hertz curve is also given. Points A, B, A' and B' are experimental data obtained with a steel cylinder in contact with a soft natural rubber surface, for four forces per unit axial length.

In a first set of experiments, the cylinder was placed on a rubber strip, $l_A = 18$ mm wide, the cylinder axis being perpendicular to the length of the strip, so that the force applied per unit axial length was $P_A = W/l_A = 6.81$ N/m. In this case, the width of the contact area was equal to $2b_A = 732\,\mu$m. If the cylinder was placed on a more narrow strip, $l_B = 10$ mm wide, corresponding to the linear force $P_B = l_B = 12.25$ N/m, the contact area was $2b_B = 816\,\mu$m wide. From the two previous forces and the two associated widths of contact areas, the Dupré energy of adhesion has been calculated by using Eqs. (3) and (4). It has been verified that its average value, $\bar{w} = 121$ mJ/m^2, was in good agreement with the value which could be deduced, using the Johnson *et al.* theory,[5] from the different radii of the equilibrium contact areas between the same rubber sample and a spherical punch submitted to several normal loads.

Thanks to the value of the Dupré energy of adhesion, it is possible to normalize experimental data by using Eqs. (8) and (9) and to plot it on Figure 1. The two corresponding points (points A and B) fall perfectly on the theoretical curve (heavy line).

In a second set of experiments, the steel cylinder was placed **under** the horizontal rubber strip, 18 mm wide, so that the force per unit axial length was equal to $P_{A'} = -P_A = -6.81$ N/m. When the equilibrium conditions were reached, the average value of the width of the contact area, calculated between 10 tests, was equal to $2b_{A'} = 375 \pm 5\,\mu$m. The corresponding normalized value is illustrated by the point A' on Figure 1; this point falls in the immediate vicinity of the theoretical curve. Moreover, it has been verified that when the steel-cylinder was placed under the 10 mm width rubber strip, so that the linear force was $P_{B'} = -P_B = -12.25$ N/m, the contact area decreased leading to a complete separation and to the fall of the cylinder (Point B' on Figure 1). Indeed, because $P_{B'}$ was more negative than the elastic adherence force $P_c = -7.75$ N/m for the same environmental conditions, there did not exist an equilibrium contact area. Thus, the validity of the model proposed for describing the adhesive contact between a rigid cylinder and an elastic half-space is confirmed.

The evolution of the contact area, when the force applied per unit axial length is modified, is illustrated on Figure 2 which represents, in reduced coordinates, the strain energy release rate variations according to the half-width b of the contact area, for several

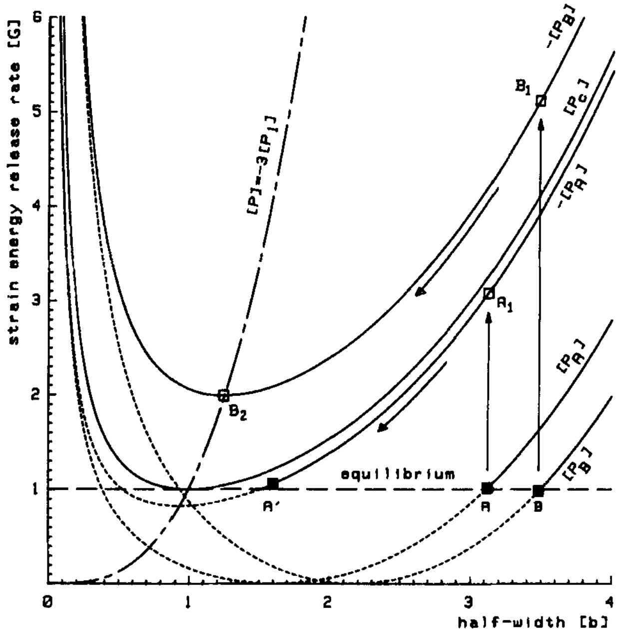

FIGURE 2 The strain energy release rate G variations according to the half-width of the contact area b, for several forces P per unit axial length, in reduced coordinates. Horizontal line $[G]=1$ represents the equilibrium curve; curves $[P]$ show the variation of G with respect to b at fixed force P.

imposed linear forces P, by using Eq. (12). The equilibrium curve is the horizontal dotted line $[G]=1$. If, starting from the equilibrium state under the force P_A when the steel-cylinder is in contact upon the horizontal rubber strip 18 mm wide (point A on Figures 1 and 2), the substrate is abruptly turned over so that the cylinder becomes in contact **under** the rubber surface, the equilibrium is disrupted because the strain energy release rate reaches an instantaneous value greater than w corresponding to the new applied force $P_{A'} = -P_A$ (point A_1, on Figure 2). This evolution at constant area of contact (part AA_1) illustrates the elastic response of the system. Then, as $G > w$, the crack tip moves forward and a decrease in contact area is observed with a continuous decrease in crack propagation speed, for $(\partial G/\partial A)_p > 0$, under the fixed force $P_{A'}$, along the corresponding curve (part A_1A'), until a new equilibrium state is reached (point A' on Figures 1 and 2).

If, starting from the equilibrium state under the force P_B (point B on Figures 1 and 2) when the cylinder is in contact upon the horizontal rubber strip 10 mm wide, one executes a similar "turn over" test, the behaviour is quite different after the elastic response of the system (part BB_1 on Figure 2) has occurred. Indeed, the new applied force $P_{B'} = -P_B$ is more negative than P_c, so that the curve, along which the force $[P_{B'}]$ remains constant, does not intersect the equilibrium curve. There is first observed a decrease in contact area with a continuous decrease in crack propagation speed until G reaches a minimum value (part B_1B_2 on Figure 2), then, an increase in propagation rate occurs, as G increases, leading to complete separation and the fall of the cylinder (point B' on Figure 1). The boundary between deceleration and acceleration stages, which corresponds to G minimum values, *i.e.* $(\partial G/\partial A)_p = 0$, is also recorded on Figure 2. It can be reasonably predicted that the detachment of a heavy cylinder directly begins at the acceleration stage.

ROLLING

In a third set of experiments, the substrate overlapped by the rubber strip was inclined. In this case, it is well known that a rigid cylinder rolls at slow and constant velocity, due to the joint effects of molecular attraction forces and viscoelastic properties of the rubber. The edges of the contact area, perpendicular to the rolling motion, can be seen as two crack tips that propagate in the same direction and with the same velocity: a closing crack in the front region, and an opening crack at the rear where the greater part of the energy is dissipated. The rolling resistance, which arises from the $\pi/2$ peeling force at the trailing edge of the contact area,[7,8] is equal to:

$$F = W \cdot \sin\beta$$

where β is the incline angle of the substrate with respect to the horizontal, and the corresponding strain energy release rate is given by:

$$G^* = P \cdot \sin\beta \tag{13}$$

where $P = W/l$ represents the applied force per unit axial length.

As observed in numerous other experiments, the increase in β from 0 to 90° increases F and G, so that the rolling speed increases.

The new result is that if the weight of the cylinder is smaller than the absolute value of the elastic adherence force defined by Eq. (6): firstly, the cylinder rolls **under** the inclined rubber strip and, secondly, the rolling speed takes the same value as if the cylinder is in contact **upon** the same inclined rubber surface. Experimental data obtained with a steel cylinder, weighing 122.5 mN and 20 mm long, are shown on Figure 3 (open and filled symbols).

Using Eq. (13), it is possible to record the variations in strain energy release rate G^* according to the rolling speed v, as shown on a log-log scale on Figure 4. This diagram clearly proves that G^* is computable by Eq. (13) for every slope ($0 \leq \beta \leq 180°$) of the cylinder trajectory, so that the energy dissipated by rolling arises solely from the work of tensile stresses at the trailing edge of the contact area, independent of the weight of the cylinder, provided that this weight is smaller than the absolute value of the elastic adherence force. Moreover, Figure 4 shows that G varies as a power function of the crack propagation speed, which corroborates a previous result.[1] In the present study:

$$G^* = k \cdot v^{0.55} \tag{14}$$

with $k = 96.484$ in SI units. So, using Eqs. (13) and (14), one can

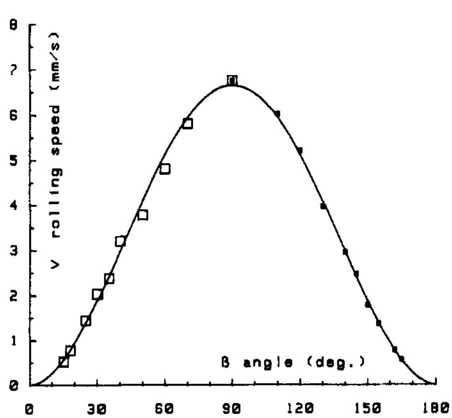

FIGURE 3 The variations in rolling speed, v, according to the angle β of the inclined rubber surface.

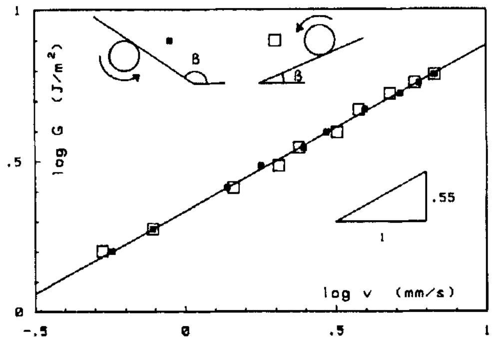

FIGURE 4 G^* strain energy release rate *vs* v rolling speed of a steel cylinder along an inclined soft natural rubber flat surface, with schematic diagrams showing a rigid cylinder in rolling contact upon and under an inclined elastic substrate (open symbols: $\beta \leq 90°$; filled symbols: $\beta \geq 90°$).

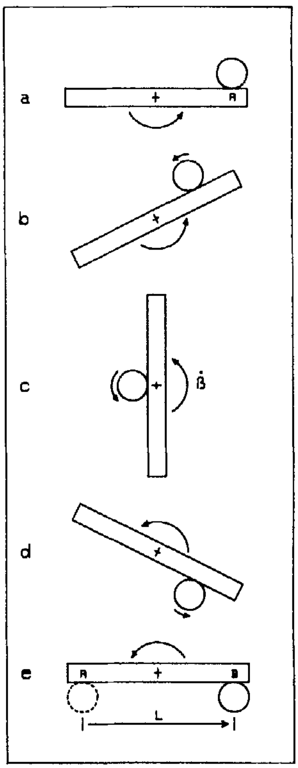

FIGURE 5 Schematic diagrams showing the rolling of a rigid cylinder in adhesive contact with an elastic substrate turned round at constant angular velocity.

compute the variations in the rolling speed v according to the incline angle β; the corresponding curve is recorded on Figure 3 (heavy line).

Due to the non-excessive value of the maximum rolling speed measured when $\beta = 90°$, a forth set of experiments was carried out in which the incline angle β of the substrate was linearly varied with time, using an electric motor imposing a constant half-turn (β varying from 0 to 180°) was measured, as schematically described on Figure 5. Several slow angular velocities were tested and Figure 6 illustrates the variations in rolling distance L according to the speed of rotation $\dot{\beta}$ (dotted points). As expected, the rolling distance decreases with an increase in rotation velocity.

The strain energy release rate G^* being calculated by using Eq. (13), we have simulated the rolling distance L variations *vs* the angular velocity $\dot{\beta}$, by means of a numerical integration of the kinetic Eq. (14). The corresponding computed curve, presented on Figure 6 (heavy line), appears to be in very good agreement with the experimental data. These results allow one to explain the following attractive experiment: when the substrate overlapped by

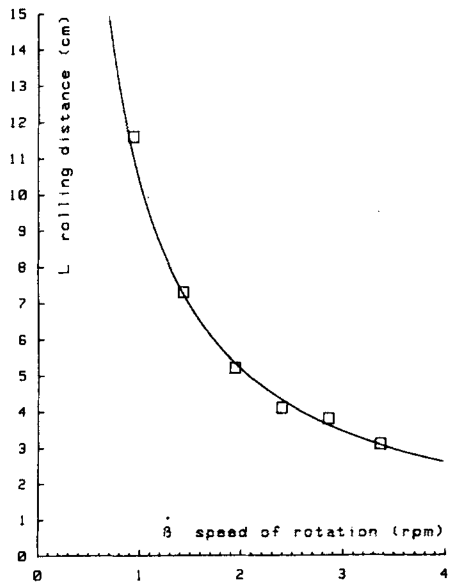

FIGURE 6 The rolling distance L variations according to the angular velocity $\dot{\beta}$ (dotted points: experimental data; heavy line: computed curve).

the rubber strip is turned round continuously, the steel-cylinder rolls alternately upon and under the rubber surface, unceasingly without falling down, provided that the length of the strip is greater than the rolling distance L corresponding to the imposed angular velocity.

CONCLUSIONS

Fracture mechanics concepts are applied to the study of the adherence of a rigid cylinder in contact with the flat surface of a half-space elastomer sample. It is shown that the cylinder remains indefinitely in contact under the surface when its weight is smaller than the absolute value of the elastic adherence force. This force depends on the cylinder size (length and radius) and on the mechanical surface properties of the elastic solid. When these conditions are met, it is shown that the cylinder rolls under an inclined rubber substrate, with the same rolling speed as if it is applied upon the same inclined surface, because the energy dissipated by rolling arises solely from the work of tensile stresses at the trailing edge of the contact area. Experiments carried out using a rubber substrate which continuously turns round at constant angular velocity, have demonstrated that the cylinder rolls alternately upon and under the surface, unceasingly without falling down, provided that the rubber sample has an adequate length.

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